

Chapter 1: Algebra

Factorizing non-monic trinomials

A non-monic quadratic is where the x^2 term has a coefficient of more than 1. Non-monic quadratic composition:

$$(ax + b)(cx + d)$$

$$x^2 \text{ terms: } ac x^2$$

$$x \text{ terms: } (ad + bc)x$$

$$\text{constant terms: } bd$$

Splitting Method:

Worked Example

$$5x^2 + 11x - 12 = 0$$

First, we multiply the x^2 term with the constant term

$$5 \times 12 = 60$$

Then, we find the factors of this product that can be summed to form the x term

$$1 \times 60, 2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10$$

Next, we use these factors to split the x terms

$$5x^2 + 15x - 4x - 12 =$$

Next is the most important part, we split the equation in half and factorize the 2 halves

$$5x^2 + 15x \mid -4x - 12$$

$$5x(x+3) \mid -4(x+3)$$

Two terms should end up the same, forming the first bracket. While the terms outside the brackets form the second bracket as follows:

$$(x+3)(5x-4)$$

Simplifying algebraic fractions

An algebraic fraction is a fraction in which the numerator and denominator are algebraic expressions. These can be simplified by cancelling out common factors in both the numerator and denominator

Worked example

Simplify $\frac{4x^2 - 16x}{6x - 24}$

To simplify the fraction, we want to cancel any common factors in the numerator and denominator. To find these common factors, we first want to factorize the expressions in the numerator and denominator.

Factorizing numerator and denominator $\frac{4x^2 - 16x}{6x - 24} = \frac{2x(2x - 8)}{3(2x - 8)}$

Cancelling out $2x - 8$ from the numerator and denominator $= \frac{2x}{3}$

Since there are no more common factors in between $2x$ and 3 , we can conclude that:

$$\frac{4x^2 - 16x}{6x - 24} = \frac{2x}{3}$$

4 Operations with algebraic fractions

When completing the 4 operations with algebraic fractions, the process is the same as with numeric fractions.

Multiplication:

Algebraic fraction can be multiplied by multiplying the numerators together, and then the denominators together. The fraction can then be simplified using common factors.

Worked example

Simplify $\frac{2y}{5} \cdot \frac{3}{m}$

Multiplying Numerators and denominators $\frac{2y \cdot 3}{5 \cdot m}$

Simplifying algebraic products $\frac{2y \cdot 3}{5 \cdot m} = \frac{6y}{5m}$

Since the numerator and the denominator have no common factors, this is the simplest form of our answer

Division:

Again, the process for dividing algebraic fractions is the same as for numerical fractions; flipping the second fraction and then multiplying the numerator and denominator

Worked example

Simplify $\frac{m}{3} \div \frac{5}{x}$

Flip the second fraction $\frac{m}{3} \div \frac{x}{5}$

Multiply numerators and denominators $\frac{m}{3} \cdot \frac{x}{5} = \frac{m \cdot x}{3 \cdot 5}$

Simplifying algebraic products $\frac{mx}{15}$

Since the numerator and the denominator have no common factors, this is the simplest form of our answer

Addition & Subtraction

When adding and subtracting fractions, the most important things to keep in mind are that the denominators need to be equal, and the fractions need to be equivalent if edited (numerator and denominator must be multiplied by same amount)

Worked example 1

Simplify $\frac{2y}{3} + \frac{5}{6}$

Find the lowest common multiples of the denominators and multiply each of them as needed to make them equal:

$$\frac{2y \cdot 2}{3 \cdot 2} = \frac{4y}{6}, \frac{5 \cdot 1}{6 \cdot 1} = \frac{5}{6}$$

Now that the denominators are equal, the numerators can be added together:

$$\frac{4y}{6} + \frac{5}{6} = \frac{4y + 5}{6}$$

The same applies if we switch the operation to subtraction, subtracting the numerators when the denominators are equal:

$$\frac{4y}{6} - \frac{5}{6} = \frac{4y-5}{6}$$